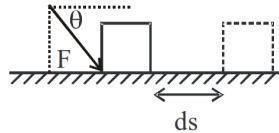


WORK, POWER, ENERGY

Work : Work is done by the force if point of application of force is displaced.



Work done by the force is the dot product of force and displacement or work done is the product of magnitude of displacement and component of force along displacement.

$$dw = \vec{F} \cdot d\vec{s} = F ds \cos \theta = (F \cos \theta) ds$$

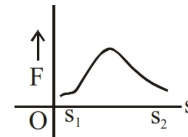
$$dw > 0 \text{ if } 0 \leq \theta < 90^\circ$$

$$dw < 0 \text{ if } 90^\circ < \theta \leq 180^\circ$$

$$dw = 0 \text{ if } \theta = 90^\circ$$

Work done by variable force for displacement S_1 to S_2 is given by

$$w = \int_{S_1}^{S_2} F \cdot ds \quad ; \quad F \text{ is force along displacement}$$



Work done by the variable force is the area below the curve drawn between component of force along displacement and magnitude of displacement.

The S.I. unit of work is joule others are erg, ev, kwh.

- 1 joule = 10^7 erg
- 1 erg = 10^{-7} joule
- 1 ev = 1.6×10^{-19} J
- 1 kwh = 3.6×10^6 joule
- 1 joule = 6.25×10^{12} Mev

Power : The rate of doing work is power

$$P = \frac{\vec{F} \cdot d\vec{x}}{dt} = \frac{dw}{dt} = \frac{dk}{dt}$$

$$P = \vec{F} \cdot \vec{v} = \text{instantaneous power delivered}$$

Unit of power is joule sec^{-1} or watt.

$$1 \text{ hp} = 746 \text{ watt.}$$

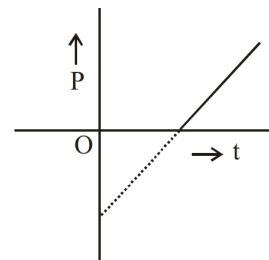
Instantaneous power delivered in case of projectile is given by

$$P = -mg(\hat{j}) \cdot [u \cos \theta \hat{i} + (u \sin \theta - gt) \hat{j}]$$

$$= -mg[u \sin \theta - gt]$$

$$= mg^2 t + (-mg u \sin \theta)$$

Power delivered at the maximum height of projectile is zero.



Energy: The ability of doing work is represented by a quantity of motion, called energy.

Mechanical Energy

(i) Kinetic energy

(ii) Potential energy

Kinetic energy : Kinetic energy of a body is the energy by virtue of its motion and it is equal to $\frac{1}{2}mv^2$; m = mass of body; v = speed of the body.

Potential energy : When work is done on a system and the system preserves this work in such a way that it can be subsequently recovered back in form of some type of energy, the system is capable of possessing potential energy. This energy is possessed by virtue of position or configuration of body or system.

Conservative and non-conservative forces: Those forces under the action of which, work done depends upon initial and final positions only and not on path followed, are known as conservative forces.

e.g. gravitational force, spring force, electrostatic force, magnetostatic force etc.

Those forces under the action of which, work done depends upon path followed are known as non-conservative forces. e.g. frictional force, viscous force, drag force etc.

Work done by conservative forces on round trip is always zero, while the work done by non-conservative force in round trip is non-zero.

Conservative forces and Potential energy: When, conservative force does positive work, the potential energy of the system decreases. Work done by conservative force is

$$W(x) = -\Delta U$$

$$\text{or } F(x) \cdot \Delta x = -\Delta U$$

$$\text{or } F(x) = -\frac{\Delta U}{\Delta x}$$

$$\text{or, } F(x) = -\frac{du}{dx}$$

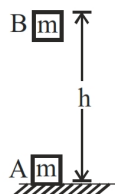
Integrating both sides, for a displacement, $x = a$ to $x = b$,

$$\text{We have : } U_b - U_a = -\int_a^b F(x) dx$$

TYPES OF POTENTIAL ENERGY

(i) Gravitational potential energy

If we lift a block through some height (h) from A to B, work is done against the gravity.



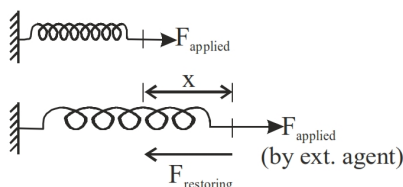
This work done is stored in the form of gravitational potential energy by external agent of the block-earth system. We can write, work done by external agent in raising the block = $(mg)h$.



- (a) If the centre of a body of mass m is raised by a height h , increase in GPE = $+ mgh$
 (b) If the centre of a body of mass m is lowered by a distance h , decrease in GPE = mgh

(ii) Elastic potential energy

When a spring is elongated (or compressed), work is done by external agent against the restoring force of the spring. This work is stored in the spring as elastic potential energy. Work done in stretching or compressing spring by a distance x is given by $= \frac{1}{2} kx^2$ and therefore elastic potential energy stored in a spring $= \frac{1}{2} kx^2$.



Work Energy Theorem

Work done by all the forces (conservative, and non-conservative, external and internal) acting on a particle or an System is equal to the change in kinetic energy.

$$W_{\text{net/all}} = \Delta K = K_f - K_i$$

If the net work done is positive, K.E. increases and if the net work done is negative, K.E. decreases.

Law of conservation of mechanical energy under the only conservative force on the system total mechanical energy is conserved i.e. $U_i + K_i = U_f + K_f$ if forces are only conservative.

As $W_{\text{all}} = \Delta K$ (work energy theorem)

$$W_{\text{ext}} + W_c + W_{\text{nc}} = \Delta K \quad \text{For } F_{\text{ext}} = 0 \text{ and } F_{\text{nc}} = 0$$

$$\text{We have } W_c = K_f - K_i$$

$$-(U_f - U_i) = K_f - K_i$$

$$\Rightarrow U_i + K_i = U_f + K_f \quad \text{Mechanical Energy conserved}$$

$$W_{\text{ext}} + W_{\text{nc}} = (K_f + U_f) - (K_i + U_i) = \text{changing mechanical energy}$$

Law of conservation of linear momentum

For a system if there is no external force then linear momentum remains conserve.

Even if there is an external force but linear momentum is conserved along the line perpendicular to the total external force.